

Inhomogeneity and the Post-Inflationary Universe

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Abstract. We discuss the interaction between perturbations in the inflaton and the background during the preheating phase in simple inflationary models. By numerically solving the Einstein field equations we are able to assess the impact of non-linear gravitational effects on preheating, and to assess the accuracy of perturbative discussions of the preheating epoch.

INTRODUCTION

Inflation drives the primordial universe towards the very special initial state needed for it evolve into the form it is observed to have today. However, to be successful, inflation must terminate gracefully with the energy density of the inflaton field being transferred to radiation, a process known as *reheating*.

Since 1990 [1] it has been realized that reheating can be driven by nonlinear, coherent effects leading to explosive particle production via parametric resonance. The resulting distribution is far from thermal equilibrium, and the process is often called *preheating* (see [2,3], and refs. within).

The standard analytic approach to preheating is based on a Fourier expansion of the relevant fields. The equation of motion of the k -th mode has the form of a Mathieu or Lamé equation. Modes corresponding to values of k in resonance bands grow exponentially, and the precise resonance structure depends sensitively on the underlying particle physics. Because it selectively amplifies specific Fourier modes, parametric resonance implies an enhancement in the spatial variation of the fields. While the non-linear backreaction on the field evolution has been examined analytically [2,4] and numerically [5,6] the backreaction of the inhomogeneity on the underlying spacetime metric and its implications for the evolution of the universe has only recently begun to be studied [3,7–13].

We have broken new ground by tackling the back-reaction problem using the full Einstein field equations, and eschewing the use of any perturbative approximations. We show that large metric inhomogeneities can be induced by parametric resonance, so approximations to the full field equations may not adequately describe the evolution of the universe during (and after) preheating. At present, our

principal simplifying assumption is that the inhomogeneity lies in a single spatial direction, which allows us to work with a $1 + 1$ dimensional system of partial differential equations.

Here we briefly survey the growth of metric perturbations during preheating in an inflationary model driven by a $\lambda\phi^4$ potential. When treated analytically, the $m^2\phi^2$ model does not undergo parametric resonance during reheating [11], a result we confirmed numerically [3]. However, $\lambda\phi^4$ possesses a single resonance band, and when we examine the evolution of modes within this band we see observe strong amplification of the corresponding metric perturbations. This work will be more fully described in our forthcoming paper [13].

METRIC AND INITIAL CONDITIONS

We have assumed that the universe has a planar symmetry, so the metric functions depend only on t and z , and are independent of x and y . In [3], we studied reheating after $m^2\phi^2$ inflation using the metric

$$ds^2 = dt^2 - A^2(t, z) dz^2 - B^2(t, z) (dx^2 + dy^2), \quad (1)$$

which describes an inhomogeneous universe in which the $dx^2 + dy^2$ sections have zero spatial curvature. In principle, we could retain this metric for the $\lambda\phi^4$ case, but it is advantageous to work in conformal-like co-ordinates analogous to those which simplify the homogeneous system [4,13]. Specifically, by transforming t and z to η and ζ we write equation (1) as:

$$ds^2 = \alpha^2(\eta, \zeta) (d\eta^2 - d\zeta^2) - \beta^2(\eta, \zeta) (dx^2 + dy^2). \quad (2)$$

In practice we use the co-ordinate transformation to compute A and B from equation (1), as well as α and β , as we evolve the system numerically. The equations of motion will be given in [13], and the numerical techniques used to solve them are similar to those we used in [3]. We focus on initial configurations where a single mode (when viewed in the perturbative context) is excited. We choose $\alpha(0, \zeta) = \beta(0, \zeta) = 1$, $\phi(0, \zeta) = \phi_0$ and $\alpha_{,\zeta}(0, \zeta) = \beta_{,\zeta}(0, \zeta) = \phi_{,\zeta}(0, \zeta) = 0$. The constraints are solved if

$$\alpha_{,\eta}(0, \zeta) = \frac{\kappa^2}{2C} \left(\frac{\phi_{,\eta}^2}{2} + V(\phi_0) \right) - \frac{C}{2}, \quad \beta_{,\eta}(0, \zeta) = \sqrt{\frac{\kappa^2}{3} \left(\frac{\langle \phi_{,\eta}^2 \rangle}{2} + V(\phi_0) \right)} = C. \quad (3)$$

Our choice of C ensures $\langle \alpha_{,\eta}(0, \zeta) \rangle = \langle \beta_{,\eta}(0, \zeta) \rangle$, where $\langle \cdots \rangle$ is a spatial average. The actual inhomogeneity is injected through the inflaton kinetic energy,

$$\phi_{,\eta}(0, \zeta) = \Pi + \epsilon \sin \left(\frac{2\pi k \zeta}{Z} \right), \quad (4)$$

where Z is the length of our “box”, and Π is the average initial velocity. Since both t and z are transformed, slices of constant η in the conformal frame are not mapped directly to slices of constant t in the physical frame. The initial density perturbation is on the order of ϵ^2 .

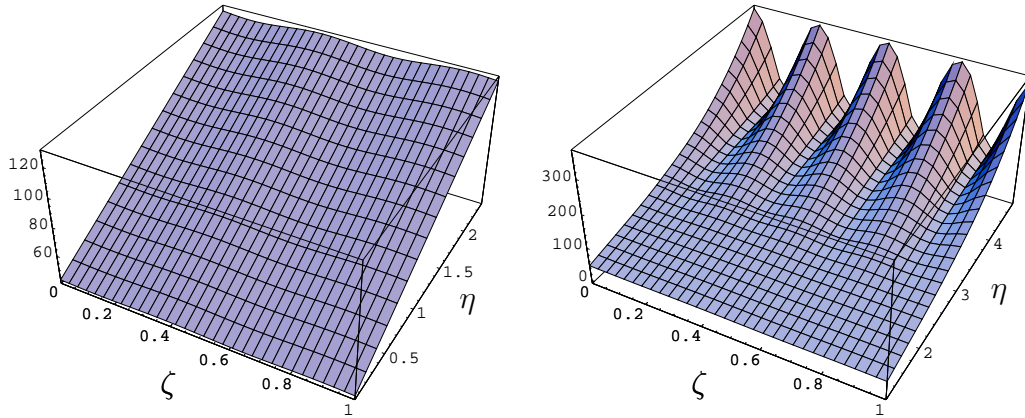


FIGURE 1. The evolution of A (the g_{zz} component of equation (1)) is plotted as a function of η and ζ . The left panel shows the evolution of a mode with k slightly too large to be in resonance, while the right hand case undergoes resonance, and significant inhomogeneity is generated. The units are arbitrary. The initial perturbation is $\epsilon^2 = 10^{-6}$, and the simulations begins at the end of inflation.

RESULTS

When we examined reheating after $m^2\phi^2$ we confirmed that there is no significant resonant amplification, and that the perturbative analysis is valid [3]. However, after $\lambda\phi^4$ inflation there is a single, narrow resonance band, and the scaling properties of the solution ensure that modes which are initially in this band remain there indefinitely.

Fig. 1 shows the evolution of the metric component A for two different perturbations, one inside the resonance band and one outside it. A significant degree of inhomogeneity is generated in the metric by the resonance in the fields. While a simple analytical treatment suggests that resonance lasts indefinitely, the backreac-

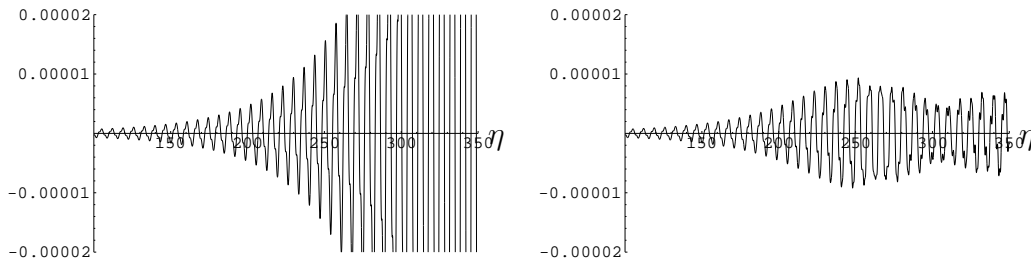


FIGURE 2. The Fourier mode of Φ , the metric perturbation, for a resonant mode is shown: the left panel gives the perturbative result, while the right panel shows the evolution of the mode derived from the full nonlinear analysis.

tion from the nonlinear field evolution eventually halts the resonant amplification. This feature is apparent in our simulation, and is illustrated in Fig. 2, where we plot the metric perturbation, Φ [14]. The termination of resonance coincides roughly with the comparatively sudden growth of the metric perturbation seen qualitatively in the right panel of Fig. 1.

If we solve for an inhomogeneous field, ϕ , while averaging over the spacetime background (corresponding to numerical treatments which include the nonlinear field equations, but assume that the metric is unperturbed) the subsequent evolution of Φ differs from that which we obtain in the presence of the gravitational backreaction. Comparing the two results will allow us isolate the contribution from the inhomogeneous parts of the metric to the overall evolution. We are also extending this work to more general initial perturbations than equation (4), so we can include interactions between different modes and study non-perturbative effects such as the formation of primordial black holes after inflation [13].

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